Methods for Estimating Lens Thickness

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BACKGROUND

Often, it is beneficial for the eyecare professional to predict the finished thickness of a pair of spectacle lenses. Determining the change in thickness that results from the patient's use of a different frame or lens style is a common example. Patients investing a considerable amount of money into thinner and lighter lenses want to know just how thin their new lenses will be. This is a question that often strikes fear in the hearts of unprepared opticians. Those armed with the knowledge required to provide the answer, however, quickly earn the respect of their patients. Even these opticians face a greater challenge today, since the advent of newer high-index lenses.

REVIEWING SURFACE GEOMETRY

The purpose of this article is to present the procedures and formulas necessary to calculate the thickness of an ophthalmic lens. We will use relatively precise equations at first, and then go on to develop simpler expressions from these that can be utilized for estimating the *approximate* thickness. It will be shown that these simplified methods will provide reasonably accurate values for your patient, without the need for tedious, time-consuming mathematics.

The first step towards determining the thickness of a given lens is developing an understanding of the relationship between surface power, diameter, and thickness. The thickness, or depth, of a surface curve at a given diameter (d) is known as its **sagitta** (s), or simply **sag**. The sag of a curve is shown in Figure 1.

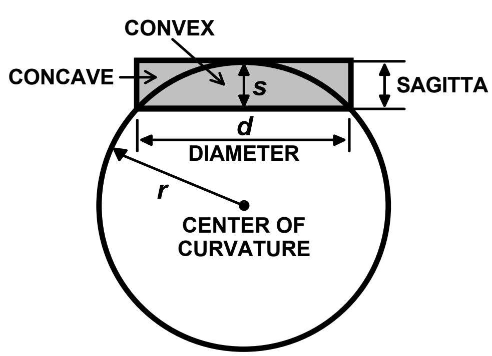

Figure 1. The sagitta of a lens surface.

For a given convex or concave curve, described by a radius of curvature r, the sagitta s can be found using the Pythagorean Theorem,

$$\left(\tfrac{1}{2}d\right)^2 + (r-s)^2 = r^2$$

$$s = r - \sqrt{r^2 - \left(\tfrac{1}{2}d\right)^2}$$

But first, we have to determine the radius of curvature. For a given surface power F_S and index of refraction n, the radius of curvature r can be found as

$$r = \frac{1000(n-1)}{F_S}$$

That sure is quite a bit of math, though! If we assume that s will be relatively small compared to r, we can simplify the sag formula and substitute the last equation for r to give us this relatively accurate equation,

$$s = \frac{\left(\tfrac{1}{2}d\right)^2 \cdot F_S}{2000(n-1)}$$

Ignore the (±) sign of the surface power (F_S); use only the absolute value for the sagitta. Stronger surface powers produce shorter radii of curvature. Hence, for a given diameter, the sag is directly proportional to the surface power and will increase as the power of the surface increases.* Further, as most opticians certainly know, the sag will also increase as the diameter of the lens increases, as shown in Figure 2.

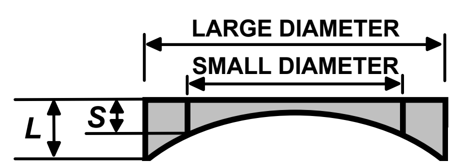

Figure 2. Diameter versus sagitta.

Now that we know how to calculate the thickness of a surface, we need to consider the form of the entire lens. Most modern lenses are **meniscus** in form, having convex front curves and concave back curves.

*If the exact formula is used, it can be shown that the sag of a curve actually increases slightly faster than its power; this is especially true for larger diameters.

If the dioptric value of the front curve is greater than the value of the concave back curve, the lens will be *positive* (plus) in power. Similarly, if the dioptric value of the back curve is greater than the value of the convex front curve, the lens will be *negative* (minus) in power. Because these lenses have *two* surface curves, we need to consider the sag of both the front curve (s_1) and the back curve (s_2) for determining thickness.

Generally, we are concerned with finding the *maximum* thickness of the lens. This will be the *center* thickness of plus lenses and the *edge* thickness of minus lenses. These lenses are often produced with a certain amount of *minimum* (or additional) thickness, as well. Therefore, in addition to the thickness of each curve, we also need to add additional *edge* thickness for plus lenses (the thinnest point of the lens) and additional *center* thickness for minus lenses (also the thinnest point of the lens). Figure 3 depicts the factors affecting the final center thickness of a meniscus plus lens.

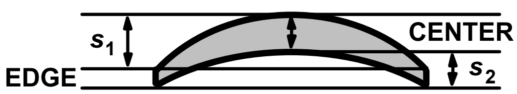

Figure 3. A meniscus plus lens.

To determine the final center thickness of a plus lens, use

$$\text{Center} = s_1 - s_2 + \text{Edge}$$

To determine the final edge thickness of a minus lens, use

$$\text{Edge} = s_2 - s_1 + \text{Center}$$

Yes, this is also a bit complex for a world demanding fast answers. When dealing with spectacle lenses of low-to-moderate power and reasonable diameter, however, we can further simplify the process by ignoring the surface curves and form of the lens altogether! This is simply an extension of our earlier sagittal approximation, which says that the sag of a curve will be directly proportional to its power. So, how is this possible?

This is possible because the surface powers of a lens (F_1 and F_2) must vary at the same rate to provide a given lens power F, so that*

$$F = F_1 + F_2$$

*This relationship holds true for *thin* lenses, and is in line with our approximation.

As a consequence, the sags of each curve must also vary at the same rate. Therefore, the difference between the sags will remain constant as the surface powers change.[1]

To visualize this concept, consider the form of the lens as being *flat*, so that the lens power is produced by one surface curve with a single sagitta. The flat plus lens will have a convex front curve and a plano (flat) back curve, while the flat minus lens will have a plano front curve and a concave back curve. With the use of our approximation, the difference between the sags of the front and back curves will remain constant.

At this point, we merely have to add the desired amount of minimum thickness to determine the final, maximum thickness of the lens. These simplified lenses are illustrated in Figures 4 and 5.

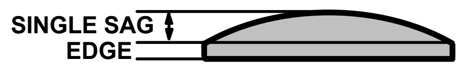

Figure 4. A plano-convex plus lens.

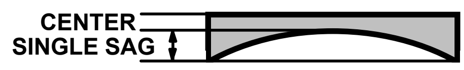

Figure 5. A plano-concave minus lens.

We will now substitute the power of the lens (F)—ignoring the (±) sign—for the surface power (F_S) in our simplified sag formula,

$$s = \frac{\left(\frac{1}{2}d\right)^2 \cdot F}{2000(n-1)}$$

And, to determine the final *maximum* thickness of the lens, use the expression

$$\text{Thickness} = s + \text{Minimum}$$

And now a word about minimum center thicknesses... The manufacturer's center thickness guidelines ensure that these lenses will have enough thickness to provide acceptable flexural stability and impact resistance.

Most minus lenses will be either surfaced to or supplied in finished form with centers between 1.0 mm and 2.2 mm, depending upon the type of lens material and design. Lenses intended for rimless or safety frames, for instance, may be slightly thicker.

To make things even easier, we can solve the equation for various values of the diameter d and the index n in advance. A table of such constants, which are called **K-values**, can be prepared and kept readily available.[2] From this point on, to approximate the maximum thickness for a given index and diameter, we need to only multiply the appropriate K-value (K) by the power F of the lens. Remember to also add the minimum thickness. A sample table of K-values is provided later on in Table 1.

$$\text{Thickness} = F \times K + \text{Minimum}$$

So far, so good—right? Up to this point, we have assumed two things: a lens power and a lens blank diameter. Obviously, the power should be known. If the diameter is unknown, a few more computations may be necessary. It is important to note that the center thickness of a *finished* plus lens is fixed with respect to the initial diameter of the lens blank. Once cast, plus lenses can only be *surfaced* to smaller diameters and thinner centers. When using finished plus lenses, the factory blank size should be utilized for determining the center thickness.

The patient's frame dimensions and interpupillary distance (distance PD) are required to determine the **minimum blank size** for a given pair of eyeglasses. The minimum blank size will be the smallest lens diameter required for a particular frame and lens combination. The dimensions of a typical spectacle frame, using the *boxing system*, are illustrated in Figure 6.

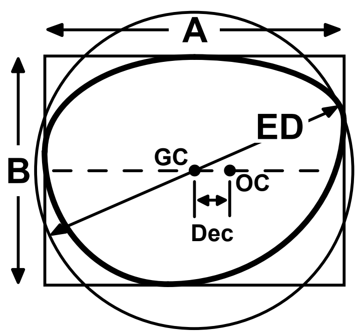

Figure 6. The Boxing System.

Once the optical center has been decentered from the geometric center, the minimum blank size (MBS) becomes equal to twice the **effective radius** (ER) of the decentered lens, which is the distance from the decentered optical center (OC) to the farthest point along the eyewire: MBS = 2 × ER.

This distance can be estimated with a simple frame measurement. An easy rule of thumb to remember for *single vision* lenses states that the minimum blank size MBS is approximately equal to the sum of the effective diameter ED of the frame and twice the required decentration (Dec), or

$$\text{MBS} = \text{ED} + 2 \times \text{Dec}$$

The effective diameter is twice the distance from the geometric center (GC) of the frame to the farthest point along the eyewire. The ED is essentially the minimum lens diameter that will completely encompass the frame, in the absence of decentration.

If we know the patient's interpupillary distance (PD), the eyesize (A) of the frame, and the width of the bridge (DBL), we can find the decentration (Dec) with

$$\text{DEC} = \frac{\text{A} + \text{DBL - PD}}{2}$$

Combining both equations gives us

$$\text{MBS} = \text{ED} + \text{A} + \text{DBL - PD}$$

It should be apparent that this rule of thumb method does *not* consider the frame shape or the angle of the effective diameter. Determining the actual effective radius of the lens will be more accurate, when possible. Once the lens has been decentered, the effective radius will usually be located along the mid-line and toward the temporal edge of the frame. For *harlequin*, *aviator*, and similar styles with high or low temporal corners, the effective radius may be displaced up or down slightly, as shown in Figure 7.

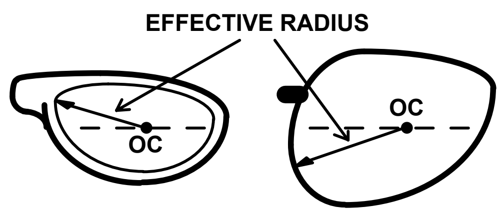

Figure 7. Some exotic frame shapes.

In general, *myopic* (nearsighted) patients are more likely to be concerned with the thickness of their lenses, since the edges of their minus lenses are quite visible to others. Let's take a closer look at calculating the edge thickness of a minus lens.

But first, we need to discuss lenses that require a cylinder component, since these lenses complicate matters somewhat. For simplicity, we are specifically concerned with the effective power of the lens through the horizontal (180°) meridian. The effective radius should be close to this meridian. For increased accuracy, you may choose to determine the effective power through the actual meridian of the effective radius.[3]

If the axis of the sphere power is close to 180°, the power of the lens through the horizontal meridian (F_{180}) is roughly equal to the *sphere* power S.

Table 1 K-values (Interpolate diameter values between the 5-mm increments)

	DIAMETER								
MATERIAL	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**
Hard Resin	0.40	0.51	0.63	0.76	0.90	1.06	1.23	1.41	1.60
Crown Glass	0.38	0.48	0.60	0.72	0.86	1.01	1.17	1.34	1.53
Spectralite®	0.37	0.47	0.58	0.70	0.83	0.98	1.13	1.30	1.48
Polycarbonate	0.34	0.43	0.53	0.65	0.77	0.90	1.05	1.20	1.37
1.6 High-index	0.33	0.42	0.52	0.63	0.75	0.88	1.02	1.17	1.33
1.66 High-index	0.30	0.38	0.47	0.57	0.68	0.80	0.93	1.07	1.21
1.7 Glass	0.29	0.36	0.45	0.54	0.64	0.75	0.88	1.00	1.14

If the axis is close to 90°, the power through the horizontal meridian (F_{180}) is roughly equal to the sum of the sphere *and* cylinder powers (S and C). For prescriptions with an oblique axis θ, between 90° and 180°, the contribution of the cylinder power must be determined using the *sine-squared rule*,

$$F_{180} = S + C \cdot \sin^2\theta$$

Now we can tell how much cylinder power C to add to the sphere power S for a given axis θ. It's really for those math buffs out there. For everyone else, get one of those handy cylinder distribution charts or learn the values for axes like 30° (25%), 45° (50%), 60° (75%), 90° (100%), 120° (75%), 135° (50%), and 150° (25%).

Keep in mind that, for a given diameter, the thinnest edge will be through the *axis* meridian of *minus cylinder* lenses, and the thickest edge will be through the *power* meridian. With these tools in mind, we can summarize the entire method for predicting the edge thickness of a minus lens.

4 Steps for Estimating Edge Thickness

1. Determine the minimum blank size using the effective diameter, eyesize, bridge, and PD measurements (or the effective radius, if this is known).
2. Determine the power of the lens through either the 180° meridian, or the effective radius meridian. Ignore the (±) sign.
3. Determine the lens material (or the refractive index), and look up the nearest K-value from the table.
4. Multiply the power by the K-value, and add the minimum thickness to this.

This method, although simplified, will provide an acceptable degree of accuracy when precision is not critical. For higher plus and minus powers of rather large diameter, the exact formulas described earlier should be employed. This is especially true for lens surfacing. The center thickness of sphero-cylindrical plus lenses may also be affected by the axis of the cylinder in some cases, but that discussion is beyond the scope of this paper. Otherwise, you can expect acceptable results for most situations.

We will conclude the discussion with an example. Your patient, Nancy Cantsee, comes in with the following prescription:

OU -3.00 DS -1.00 DC × 045

PD 62 mm

She selects a stylish round frame with a 52-mm eyesize, a 54-mm ED, and a 16-mm bridge. You are fitting her with SOLA's Spectralite® ASL finished aspheric lenses.

1. The minimum blank size needed here is approximately equal to:

$$\begin{aligned} \text{MBS} &= \text{ED} + \text{A} + \text{DBL} - \text{PD} \\ \text{MBS} &= 54 + 52 + 16 - 62 \\ \text{MBS} &= 60\text{ mm} \end{aligned}$$

2. The effective power through the 180° meridian of these lenses is:

$$\begin{aligned} F_{180} &= S + C \cdot \sin^2\theta \\ F_{180} &= -3.00 + (-1.00)\sin^2 45 \\ F_{180} &= -3.50\text{ D} \quad (\textit{Ignore} \pm \textit{sign.}) \end{aligned}$$

3. You are using Spectralite®, which has a K-value of 0.83 at a 60mm diameter.

4. The finished ASL lens has a 1-mm center thickness, which gives us a total of:

$$\text{Thickness} = s + \text{Minimum}$$
$$\text{Thickness} = 3.50(0.83) + 1$$
$$\text{Thickness} = 3.91\,\text{mm}$$

The final answer: 3.91 mm. It is interesting to note that our estimation is within *0.15 mm* of what the lens would actually produce! However, greater errors may be encountered depending upon the lens form, power, and finished diameter.

It should be kept in mind that the approximations described here are not entirely accurate for aspheric surfaces. For instance, the geometry of the ellipsoidal aspheric surface described in Figure 8 will afford a slightly smaller sag value than a comparable spherical surface of the same diameter. Consequently, the sagitta calculations previously described should only be used for an estimation of thickness.

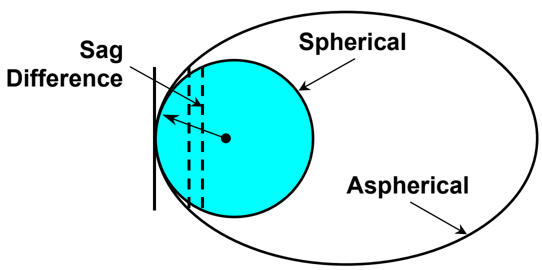

Figure 8. The difference in sag values between a spherical and a comparable aspheric surface.

Thickness is obviously a very important aspect of ophthalmic lenses. However, consideration should also be given to other factors that contribute to the overall cosmesis of a pair of eyeglasses. For instance, materials with lower densities will be lighter in weight, and flatter base curves will reduce the bulbous appearance of the lenses. Although a discussion of these factors would be out of the scope of this article, they should certainly be given equal attention when fitting eyewear.

References

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3. Chase, G. *Decentration and Minus Lens Thickness.* Washington, D.C.: Optical Laboratories Association, 1977. Page 2